



Energy system challenges and their mathematical solutions

Results of a brainstorm

How mathematics contributes to the energy system transition

Our energy system needs to change from fossil-based to more sustainable energy based. This raises significant challenges, such as enhancing the resilience of the energy system against future (price) perturbations and further optimizing the use of the existing capacity.

This whitepaper aims to identify major areas where advances in mathematics and computing can lead to either immediate or long-term solutions to the challenges around energy systems in the Netherlands and beyond. It is the outcome of a half-day brainstorm session of mathematicians with stakeholders in energy systems.

Significant improvements from current mathematical developments can be expected in the following areas, that were each identified as very important by participants from the energy sector:

- More and better tools to deal with uncertainty.
- Understand the effects of regulation.
- Improved methods for forecasting demand and (renewable) supply.
- Improved and faster DC and AC load flow solvers.

The big moonshot goal is to have a more integral method for long-term planning that covers all the aspects involved and that is nevertheless computationally feasible.

This whitepaper identifies the mathematical disciplines that can contribute to these challenges and mentions some current mathematical developments that could open new avenues to solutions.

This whitepaper is a starting point rather than a definitive overview of the possibilities for mathematicians to address challenges around our energy system. The fact that so many avenues for progress were identified in just half a day testifies to the potential contribution that mathematics can make to the energy transition. The list of contacts at the end of the whitepaper will help in building meaningful collaborations between the energy sector and mathematicians.

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1. Introduction

Energy systems play a central role in the transition that we need to make towards renewable energy. Many challenges around energy systems require things like forecasting, optimization, planning and risk assessment, which are in essence mathematical or computational in nature. It is only natural, therefore, to apply advances in computing and mathematics to solve these challenges. This whitepaper proposes avenues to do just that.

This whitepaper is the result of a brainstorm by a group of specialists from the energy sector and a group of mathematical researchers. To deepen the results, additional conversations were held with some of the specialists and researchers.

The ambition of this whitepaper is to stimulate actual development and collaborative research by identifying key areas for joint progress. The whitepaper is intended as a basis for discussion in, for example, the top sector Energy and as an inspiration for NWO programs.

1.1. WHICH ENERGY SYSTEMS ARE WE TALKING ABOUT?

The term “energy system” is rather broad. It ranges from small scale batteries and heat pumps to distribution networks across continents. Importantly, it should be realized that modern energy systems include not only physical but also virtual components and involve both human and software actors. The brainstorming workshop concentrated on distribution networks (the ‘grid’), its design, planning and operation and the rules that govern it.

1.2. WHAT KEY CHALLENGES WERE IDENTIFIED?

Out of the many challenges, 4 key areas were elaborated:

1 Uncertainty makes planning difficult both on a strategic

and on an operational level. Changing energy systems takes years if not decades. In the meantime, much can change (like the outbreak of a war that throws a wedge in the gas supply). And then the systems also need to be able to deal with the daily uncertainty like the varying supply of wind power and solar power. Dealing with all this requires making good decisions with very incomplete information. Chapter 2 goes into this topic.

2 Local hubs can make the network more resilient and adaptive, but the local rules that apply (like who gets priority and who pays for what) need to be designed. These rules will also impact the control that network operators have. Designing functioning rules is essential in distributed energy systems. This is the topic of Chapter 3.

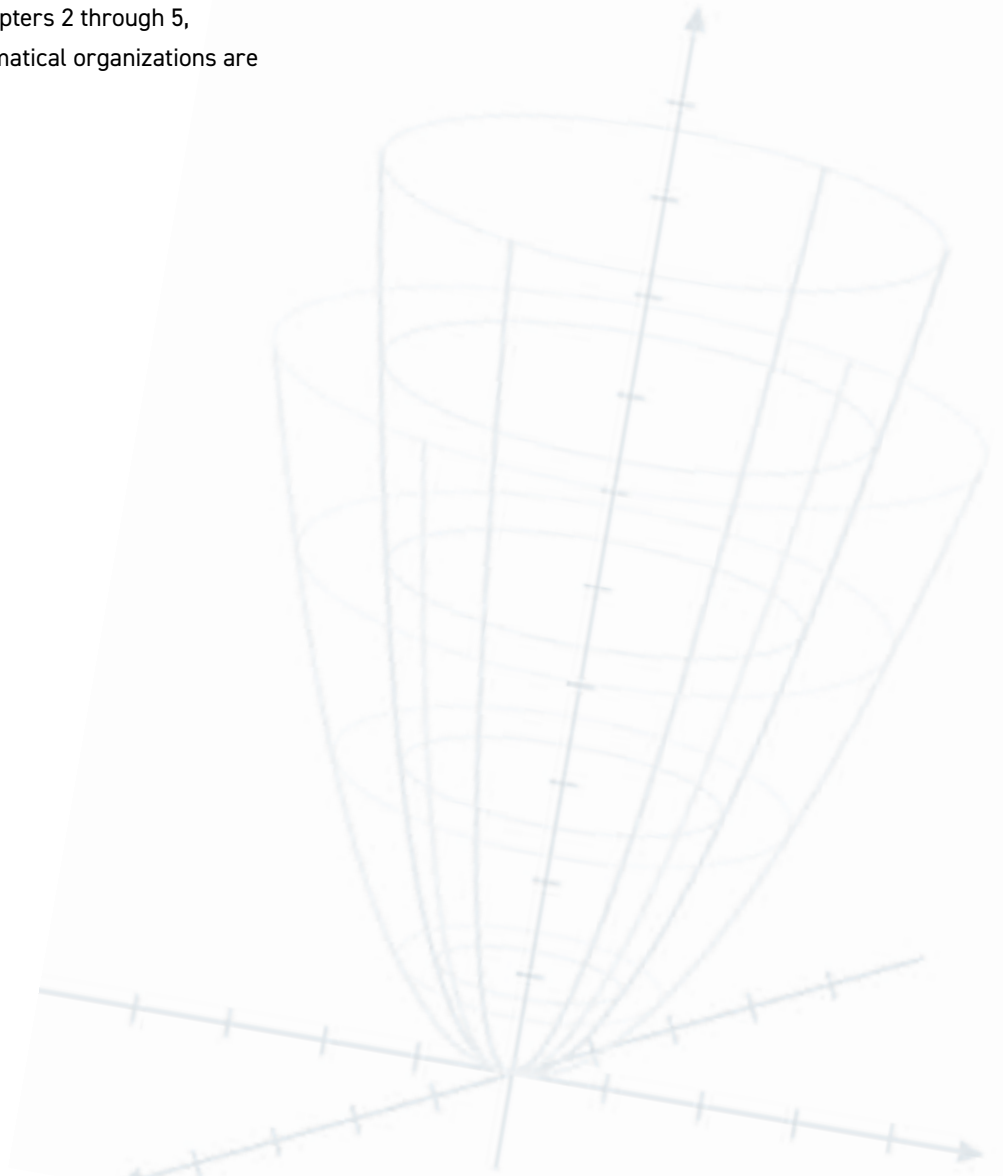
3 Energy system operators use tools to monitor, analyse and control the network. The current energy network is becoming more complex as next to traditional fossil-based there are also renewable energy sources (solar, wind, biomass) involved and the networks are becoming decentralized. As a result, also these tools need to evolve. This is elaborated in Chapter 4.

4 Long term planning is now a linear process that does not consider all the interactions between the various steps. It would be good to define and solve the overall problem, including multiple modalities (heat, electricity), decentralized networks, stochastics, behavioural models, market models and congestion. This is the subject of Chapter 5.

Cyber security was also identified as a very important topic to which mathematics can contribute a lot. It warrants a brainstorming session of its own to identify all the mathematical advances that apply and therefore this topic was put on the list for a future event.

1.3. HOW CAN MATHEMATICS CONTRIBUTE?

For each of the challenges the most relevant mathematical disciplines are given in the following chapters as were identified by the participants. This does not mean there are no other disciplines that may be able to contribute. Some current mathematical research themes that are of particular interest are mentioned. For each of the mathematical disciplines that are mentioned in Chapters 2 through 5, pointers to some appropriate mathematical organizations are given in Chapter 6.



2. Planning in the face of major uncertainty

2.1. THE CHALLENGE

Uncertainty abounds in the energy transition. We do not know how renewable energy technology will develop in the coming years. For example, what will be the effect of the electrification of trucks? Nor do we know what kind of regulation will be in place or even what the geopolitical situation will be.

Still, plans for the energy system of the future need to be made now as the implementation of these plans will typically take years if not decades. How can we assess the resilience/robustness of networks considering renewable energy sources, fluctuations (both short term and long term) in demand and supply and possible resistance from society (e.g., against small nuclear reactors)? And what approaches can enhance the resilience/robustness against future (unforeseen) shocks?

2.2. THE MATHEMATICS

There are several mathematical disciplines that apply here.

- *Uncertainty quantification: a key insight is that it is important to identify the sources of uncertainty and their nature before applying mathematical approaches. The NUSAP approach is relevant here: a notational system for the management and communication of uncertainty in science for policy.*
- *Stochastics: this field studies systems that are ruled by randomness and processes that are inherently random. Relevant subjects that are actively studied in this field are rare events, extreme values and correlated random variables, but also stochastic differential equations and agent-based models that describe how essential, physics-based processes with random components may lead to different evolutions.*
- *Optimization theory: a relevant term here is robust optimization for stakeholders, which means generating a range of scenarios and then defining no-regret actions.*



3. Designing rules that work

3.1. THE CHALLENGE

Local hubs can make the network more resilient and adaptive, for instance, an agreement between companies on who uses what amount of energy at which times to better distribute demand. The rules for each hub may vary though. These rules can be in legal terms, but also moral rules or rules that are enforced by technical means. Poorly designed rules, or rules that are perceived as unfair, may stimulate cheating and the misuse of loopholes. Therefore, local rules that apply need to be investigated and designed. These rules will also impact the control that network operators have. Designing functioning rules is essential in distributed energy systems.

3.2. THE MATHEMATICS

There are several mathematical disciplines that apply here:

- *Control theory: An interesting topic in control theory that would help here is to design control protocols in feedback form: the control protocol takes the state of the system into account and can accommodate deviations from the assumed dynamics. Both the objective and the constraints can be dynamic. Another current research topic is to look at feasible control rather than optimal control: feasible control ensures adherence to constraints, while optimal control seeks to optimize a specific objective function.*
- *Systems theory: In this field mathematicians developed a concept called "Contract", which can help model systems*

with both a high-level 'large' scale and a low-level 'small' scale. The contract describes how the high-level scale sets boundaries on the low-level scale and forces the low-level scale to operate within the boundaries.

- *Game theory: Concepts like cooperative/non-cooperative gaming, democracy versus a benevolent dictator and capacity competition are of interest here. In cooperative gaming, participants can make agreements whereas in non-cooperative gaming the participants must make decisions independently. Democracy and benevolent dictator are two models describing how decisions can be made. Capacity competition models a situation where actors compete based on the amount of capacity they can allocate. Closely related to game theory is the concept of autonomous agents, e.g., in agent-based modelling.*
- *Mathematical Optimization: Clearly, this field has many applications for energy system design and operation. In this context, an active field of research is robust optimization, taking rare events into account. Taking fairness into account is another topic that is attracting much research.*
- *Complex networks: techniques to reduce the complexity of probabilistic models help in the design of effective regulation and optimization mechanisms. Modelling approaches like agent-based modelling can combine complex network descriptions, systems theory, and game theory.*

4. Helping daily operations deal with increased complexity

Energy system operators use digital tools to monitor, analyse and control the network. As the network gets more complex due to the impact of renewables and decentralization, these tools need to evolve. Tool improvements are needed for:

- Computing and predicting the state of the network.
- Forecasting (e.g. of demand and supply, but also prices) to allow optimal control of the network.

4.1. COMPUTING AND PREDICTING THE STATE OF THE NETWORK

4.1.1. The challenge

- Currently the solvers predicting DC load flow are too slow for large networks. There is a need for solvers with a solution time that scales (almost) linearly with the number of branches in the network.
- Improved alternative solvers are needed for AC load flow that can also deal with decentralized markets, i.e., markets where decisions on generation and use are made by decentral authorities. These can be grid operators in different countries but can also be local energy hubs that form a small-scale market.

4.1.2. The mathematics

Two mathematical disciplines that apply here:

- *Numerical Analysis is working on methods to solve load flow problems faster. Among the more recent developments are the (improved) HELM method, improved Newton Raphson methods and Model Order Reduction.*
- *Machine learning (ML) is an entirely different way of modelling the problem: rather than explicitly modelling the grid, machine learning builds a model by automatically learning its behaviour from historical data or previous simulation results. Once the behaviour has been learned, the ML-model can make very fast computations of what is happening in the network. Reliability of machine learning models is likely to be an issue that warrants research.*

4.2. FORECASTING TO ALLOW OPTIMAL CONTROL OF THE NETWORK

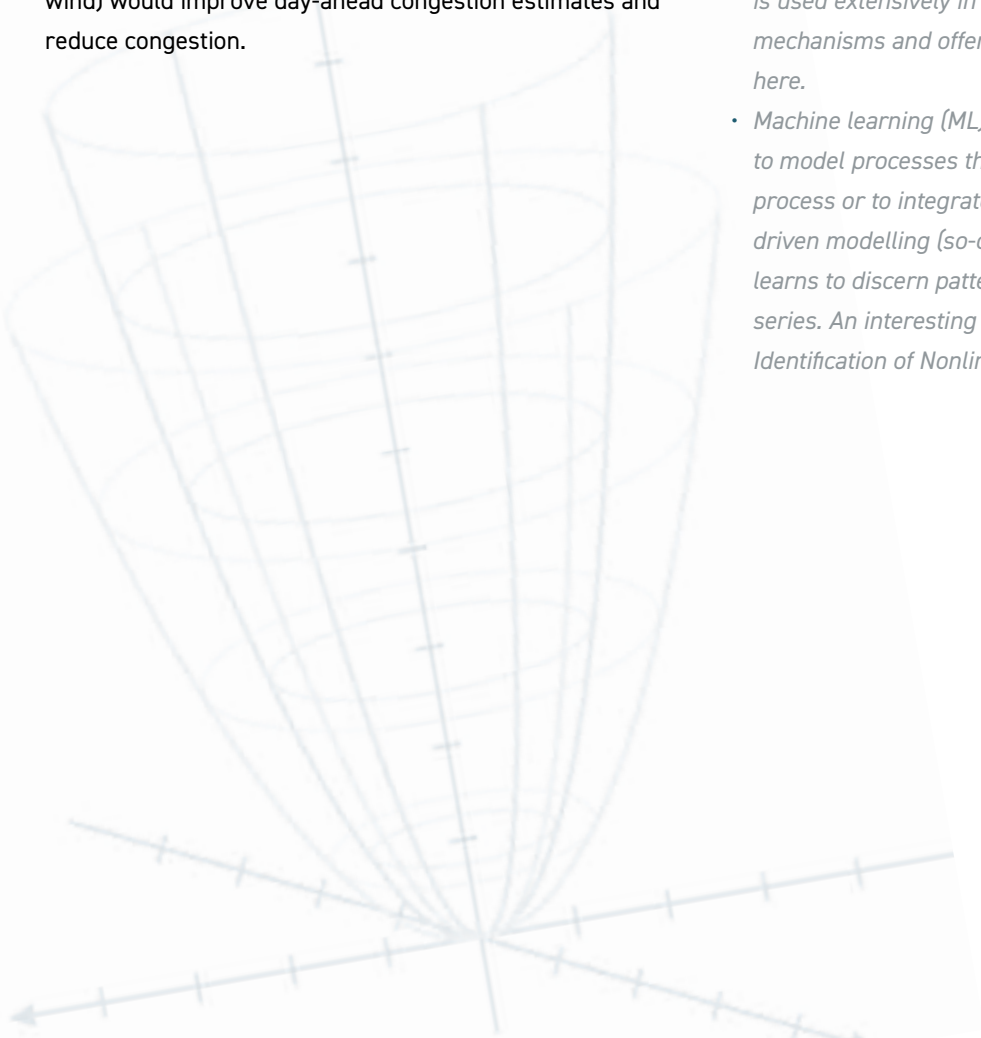
4.2.1. The challenge

- There is a need to forecast demand and supply (and hence prices) on daily and hourly basis, taking decentralized markets into account. The forecasts need to be accurate and detailed (e.g., for local markets). A topic of interest is the ability to forecast or steer the demand and supply of large players in the market. This can create opportunities for optimizing the usage of the power grid.
- Improved forecasts of renewable energy production (solar, wind) would improve day-ahead congestion estimates and reduce congestion.

4.2.2. The mathematics

The mathematical disciplines where relevant advances are being made are:

- *Statistics: this mathematics discipline studies how to best infer properties of (partially) random systems from observational data.*
- *Stochastics: this discipline studies how processes that are ruled by randomness behave, given the nature of the underlying chance processes. Stochastic differential equation modelling as used in finance is applicable here. A relevant subfield is Stochastic differential equations, which is used extensively in the study of financial markets and mechanisms and offers a rich set of methods that are useful here.*
- *Machine learning (ML): machine learning can be used to model processes that have no underlying physical process or to integrate deterministic modelling and data-driven modelling (so-called Scientific Machine Learning). It learns to discern patterns in data, in this case usually time series. An interesting recent development is the Sparse Identification of Nonlinear Dynamics (SINDY) method.*



5. The ultimate challenge: find a real trade-off of all the aspects in planning the future energy system

5.1. THE CHALLENGE

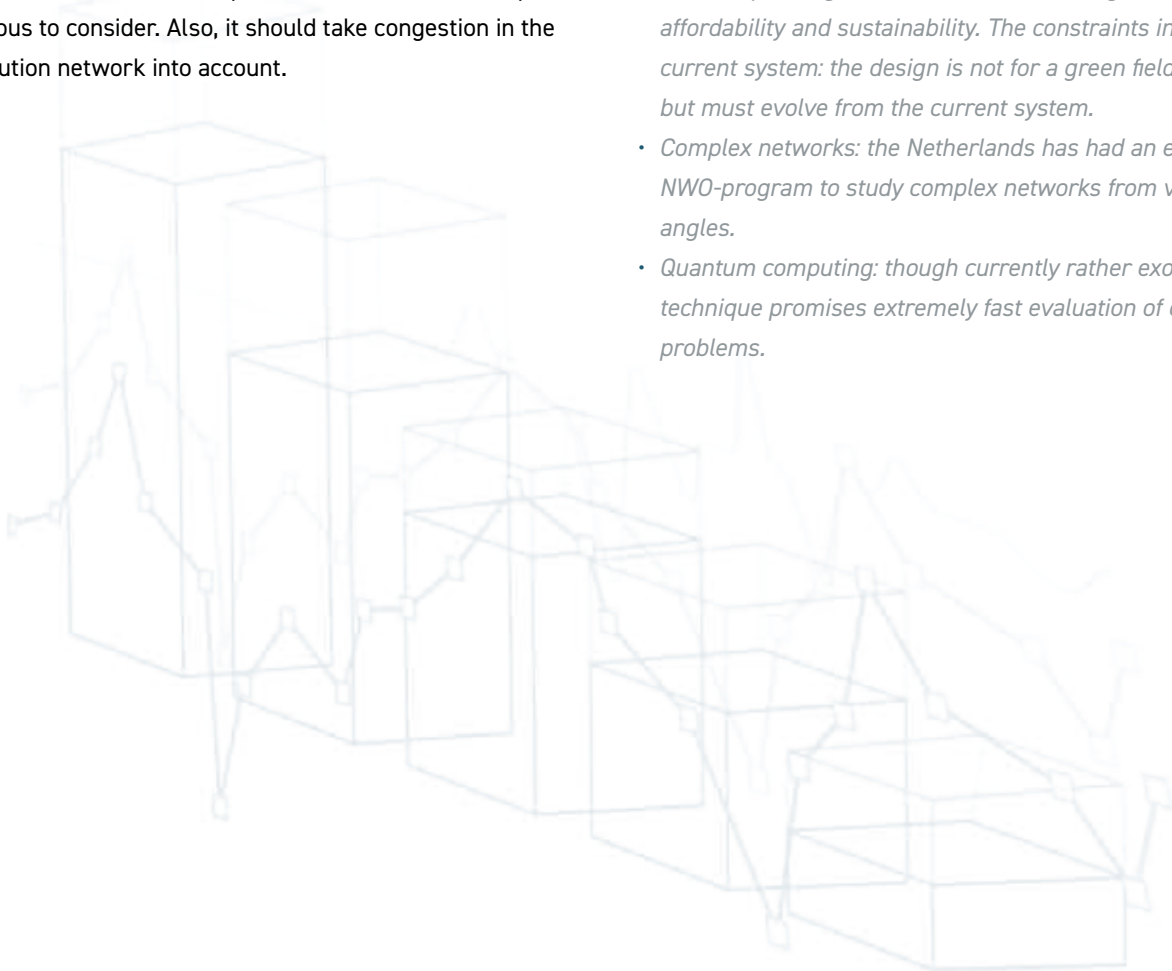
Energy system integration takes many aspects into account: sustainability, affordability, fairness, production, consumption, technical, social, economic, spatial planning. How to balance all these aspects?

Long term planning is now a very linear process that does not sufficiently respect all the interactions between the various steps. It would be good to define and solve the overall problem: including multiple modalities (heat, electricity), decentralized networks, stochastics, behavioural models, market models and politics, even as this last topic is ambitious to consider. Also, it should take congestion in the distribution network into account.

5.2. THE MATHEMATICS

There are several mathematical disciplines that apply here:

- *Mathematical modelling: A relevant concept is decision-based model selection, constructing a model for a specific purpose rather than using a general model that may be too complex for the purpose. For example, it is probably not necessary to model all the physics of a heat network in detail. Another relevant theme here is Reduced Order Modelling: reducing the size of the model by only retaining the most important features.*
- *Optimization: the challenges in mathematical terms here are the specific goals and constraints. The goals include affordability and sustainability. The constraints include the current system: the design is not for a green field situation but must evolve from the current system.*
- *Complex networks: the Netherlands has had an extensive NWO-program to study complex networks from various angles.*
- *Quantum computing: though currently rather exotic, this technique promises extremely fast evaluation of certain problems.*



6. Pointers to further information

The overview above only sketches the main connections between the challenges of energy systems and the mathematics that can solve them. This chapter offers pointers and links to organizations that can help in taking the next step.

6.1. FIND THE RIGHT MATHEMATICIANS.

To get into contact with relevant mathematicians in the Netherlands, the best way to go is through Math4NL (see math4nl.nl). This is an organization that is dedicated to finding the right mathematicians for challenges in society and industry. They can be contacted by sending an email to info@math4nl.nl.

At the moment the mathematical community is setting up mathematical contact points for each of the top sectors of the Dutch Mission-driven Innovation Policy. Please contact innovatie@platformwiskunde.nl for the latest information.

6.2. RELEVANT ORGANIZATIONS AND PROJECTS

The following organizations are doing relevant research. This is certainly not an exhaustive list, and some relevant names are bound to be missing. Math4nl can help you contact the relevant mathematical groups in this list or beyond.

- Relevant research on machine learning is taking place in the **AI for Energy Grids Lab** (see icai.ai/icai-labs/ai-for-energy-grids/).
- More general research on AI and mathematics is concentrated in the group **AI and Mathematics** (see aimath.nl).
- The Center for Mathematics and Computer Science, **CWI**, in Amsterdam is doing a lot of work on uncertainty quantification, game theory and quantum computing (see cwi.nl).
- The research school **DISC** (disc.tudelft.nl) brings together all university groups in the Netherlands that are working on Systems and Control.
- The **Energy and Industry group** of the Faculty of Technology, Policy and Management faculty at Delft University of Technology works on modelling the (transition of) the energy system.
- The **Industrial Engineering and Innovation Sciences** and **Technology, Innovation & Society** groups at the TU Eindhoven work on related topics.
- The **Korteweg de Vries institute** in Amsterdam is known for its work on stochastics (kdvi.uva.nl).
- The **MESA+** institute in Twente is doing a lot of work on optimization (see www.utwente.nl/en/mesaplus/).
- The Dutch **NETWORKS** project has done, and is still doing, a lot of research on complex networks (see thenetworkcenter.nl).
- The **VVSOR** is the Dutch organization of statisticians (see www.vvsor.nl).

7. Final remarks

This whitepaper has identified 4 main innovation directions where mathematics can have a decisive impact on challenges in energy systems. By spreading this whitepaper and the associated infographic, we hope to stimulate collaboration to take up these innovation directions.

We thank all the participants of the workshop for their contributions. It has been an interesting experience thanks to the open and constructive attitude of all. In particular, we would like to thank NWO and the Dutch Platform for Mathematics (PWN) for their valuable support.



