Stieltjesprijs academic year 2021-2022 : Report of the jury

The jury consisted of Erik van den Ban (UU), Odo Diekmann (UU, chair), Aernout van Enter (RUG), Sara van de Geer (ETH Zürich), Frans Oort (UU), Marc Uetz (UT) and Kees Vuik (TUD), with assistance from Marieke Kranenburg (UvA, secretary). A total of 74 dissertations were assessed. External advice was obtained at various stages of the assessment. After an initial selection, a shortlist of 12 dissertations was compiled. On Thursday December 8 2022 the jury met to discuss those 12 and to choose a winner.

The jury was impressed by the high level of the research performed. The quality of no less than 7 theses was such that winning the prize would be deserved. Hence difficult choices had to be made. After the first round of discussion, 4 dissertations remained, all of them of impressive quality and great originality. After an extensive second round of discussion, two dissertations remained. Both of these are of extraordinary quality, but otherwise they are rather different. So the jury was not caught in a deadlock, the consensus was that both of these should win.

The dissertation "Geometric Aspects of Linear Programming" by Sophie Huiberts (CWI, UU) offers astounding new insights in several long-standing and highly relevant problems in combinatorial optimization. One of the main results is an *effective* polynomial bound on the *expected* run time of (a new variant of) the classical simplex method for linear programming, when the problem is "smoothed" by adding small Gaussian noise to the data. The estimate is based on clever and creative use of a geometrical perspective, in combination with a sophisticated probabilistic analysis. In addition, the thesis contains powerful new bounds on the diameter of random polytopes and it introduces a new efficient scale free interior point method. (Incidentally, on October 4, 2022, the French newspaper "Le Monde" reported about her work.)

In the dissertation "Quantum information theory and many-body physics", Freek Witteveen (CWI, UvA) introduces mathematical structures motivated by potential applications of quantum computing in the simulation of quantum many-body systems. The ensuing mathematical analysis of these structures is very challenging, yet leads to novel and profound results.

The thesis combines intuitive physical motivation with rigorous mathematical formulation and analysis, drawing from and contributing to a multitude of mathematical fields. Its clarity, and the overview provided, attest to the extraordinary maturity of the author. The remarkable results have drawn international attention. According to expert referees they instigate a new research domain in quantum information theory and are bound to have a long-lasting impact.

The jury nominates both Sophie Huiberts and Freek Witteveen as winners of the Stieltjes Prize 2021-2022.

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What follows is a more detailed description of the achievements of both winners, partly based on the expert reports that were received.

In her thesis “Geometric Aspects of Linear Programming”, written at the CWI with Daniel Dadush as co-promotor and defended at Utrecht University with Gunther Cornelissen as promotor, Sophie Huiberts presents several milestone results in the theory of linear or convex optimization.

One of her contributions is a new theoretical analysis of the workhorse of commercial and academic software in optimization, the Simplex method for linear programs, invented by George B. Dantzig in 1947. This algorithm walks along neighbouring vertices of the polyhedron described by the set of feasible solutions. Even though almost all known pivot rules for deciding which path to take have been shown to have a poor theoretical behaviour in the worst-case, the Simplex algorithm has a stunningly good efficiency in practice. Sophie analyses the Simplex algorithm in the “smoothed” situation proposed by Spielman and Teng. In this situation the constraints are slightly perturbed by Gaussian noise. Spielman and Teng could derive the first polynomial upper bound on the *expected* number of pivots of the Simplex algorithm in this model. Their bound has been improved over the years. Sophie succeeds to obtain a new bound on the expected number of pivots which, in contrast to all prior works, deserves to be called “practical”. Her result rests on a totally revamped analysis of the so-called shadow bound, which counts the number of edges of a projection of the Simplex path on the perturbed polyhedron onto dimension two, together with a new variant of the Simplex algorithm that is tailored to the new shadow bound. This result alone can be seen as a masterpiece.

A second milestone result contained in Sophie’s thesis addresses the very challenging problem to estimate the combinatorial diameter of (random) polytopes, which is the maximal length of a shortest path between any two vertices. This relates to the well known Hirsch conjecture, which states that the combinatorial diameter of any polytope is bounded by m-n, when m is the number of linear constraints and n is the dimension. While this conjecture was disproved by Santos in 2012, the (updated) conjecture states that any polytope’s diameter is bounded by a polynomial in m and n. In her thesis, Sophie proves a new and almost tight bound on the diameter of a very natural class of (random) polytopes. To obtain these new results, Sophie builds on, and extends, earlier works using new techniques that combine geometric insights with probabilistic analysis in unexpected ways, leaving some of her senior colleagues baffled. Her result on polytope diameter marks a major progress in discrete geometry since about 50 years.

While these two results by themselves deserve a prize, Sophie’s thesis has two more chapters which are noteworthy. One chapter presents a new interior point algorithm for linear optimization, with computation time polynomial in n, m and the bit complexity of the constraint matrix. The question whether such an algorithm exists, with a computation time that does not depend on scaling of the columns of the constraint matrix, was open since 2003. This question now has been solved affirmatively by Sophie. A final chapter presents a simple, practically fast and easyto-implement, algorithm for convex optimization in the oracle model, inspired by the Frank-Wolfe convex combination algorithm, which compares favourably to several of the previously known techniques (such as the Ellipsoid method) in computational tests.

While one could say that many of these results are “only” improvements of worst-case analyses of variants of known algorithms in optimization, all results mark a leap forward in an area that was heavily researched in the past 50 years, and Sophie’s supervisor as well as an external reviewer are full of admiration for the mathematical depth and ingenuity of Sophies work in combining geometric insight with probabilistic analysis of algorithms. The results have been published in the proceedings of leading conferences on theoretical computer science and computational geometry (STOC and SOCG), and her work on the Simplex algorithm was recently featured in an article in the French newspaper “Le Monde”. Sophie now holds a fellowship of the Simons Foundation and works as a PostDoc at Columbia University, New York.

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In his thesis “Quantum information theory and many-body physics”, written at the CWI and defended at the University of Amsterdam with Michael Walter and Eric Opdam as promotors, Freek Witteveen identifies three main themes on the interface of quantum information theory and many-body systems: 1) the structure of ground states of many-body systems, 2) understanding the unitary quantum dynamics, and 3) the relation of quantum information and quantum gravity. Intuitive physical questions about these themes are then transformed to problems formulated in precise mathematical terms.

In part 1 a certain construction principle for ground states of quantum systems in one physical dimension is linked to the theory of wavelets (traditionally used in signal processing). On the mathematical side, new bi-orthogonal wavelet filters in one dimension are obtained, together with error estimates for so-called approximate Hilbert pair wavelets. The analysis relates to precise and physically relevant estimates of approximation errors for certain multi-point correlation functions.

Part 2 deals with the unitary time evolution of one-dimensional quantum systems, under the assumption that in a single time step locality is approximately preserved. In case of strict preservation, such systems are called quantum cellular automata. These have been classified by means of an index measuring information flow. In the thesis this index theory is extended to automata with approximate local preservation, also resulting in a classification.
Mathematically, this work incorporates and advances results about near inclusions of von Neumann algebras.

Part 3 is devoted to the theory of so-called random tensor networks, and its relation to the black-hole information paradox and the famous AdS/CFT correspondence. A toy model in terms of the mentioned networks is analyzed in a mathematically rigorous way, requiring significant new ideas from random matrix theory and smooth entropies. With this toy model Freek managed to produce rigorous mathematical results that are in line with heuristic predictions from the mentioned areas of quantum gravity.

In each of the three parts, Freek identifies challenging problems at the core of the corresponding research area, and contributes both new mathematical results and novel conceptual insights.

Freek's work pairs overview and originality with remarkable productivity. His thesis is based on four papers, three of which have been published in leading journals, including Communications in Mathematical Physics and Annales Henri Poincaré. He also had his work accepted at the conference Quantum Information Processing (QIP), which is top in the field. Parallel to his thesis he co-authored two more publications.

In October 2022 he started as a postdoc at the QMATH center of the University of Copenhagen.